Over the River and Through the Woods

4

Describing Functions

WARM UP

Does the table describe a function? Explain your reasoning.

x	y
0	1
1	2
2	3
–1	2
–2	3

LEARNING GOALS

- Analyze a problem situation using multiple representations.
- Determine characteristics of linear functions.
- Graph linear functions and describe them as functions whose graphs are straight lines.
- Identify intervals of increase, decrease, and constant values of a function.
- Define, graph, and analyze non-linear functions and give examples of functions that are not linear.

KEY TERMS

- linear function
- increasing function
- constant function
- decreasing function
- interval of increase
- interval of decrease
- constant interval
- absolute value function
- quadratic function
- cubic function

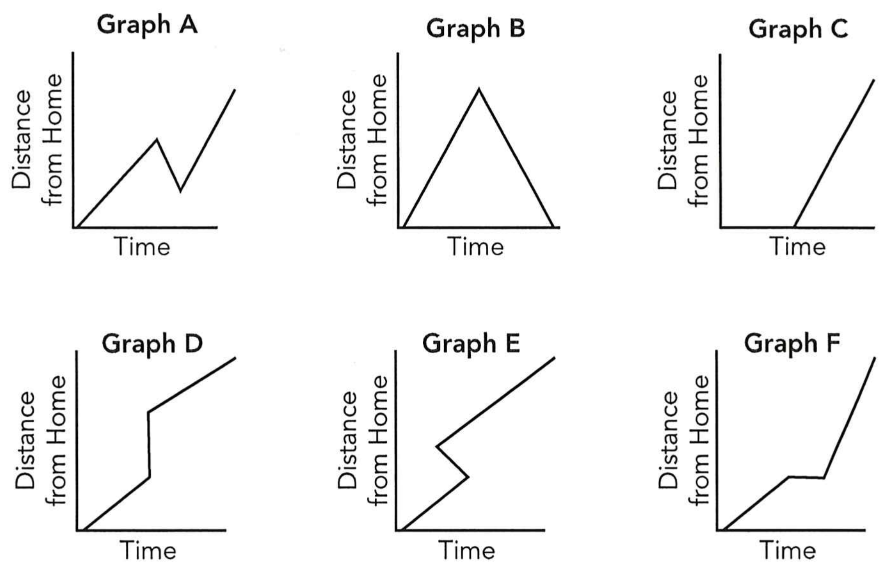
You have examined a number of different functional relationships. How are functions categorized in terms of direction and shape?

Getting Started

To Grandmother's House We Go

Little Red Riding Hood is traveling to Grandmother's house to bring her cookies and tea. However, Little Red Riding Hood often gets distracted on her way to Grandmother's house.

1. Select one of the graphs that could be her journey and describe Red's journey to Grandmother's house.



2. Which of the graphs could not be a graph of Red's journey to Grandmother's? Explain your reasoning.

3. Which of the graphs represent functions? A, B, C, F

ACTIVITY
4.1

Linear Functions

You and your friends are rock climbing a vertical cliff that is 108 feet tall along a beach. You have been climbing for a while and are currently 36 feet above the beach when you stop on a ledge to have a snack. You then begin climbing again. You can climb about 12 feet in height each hour.

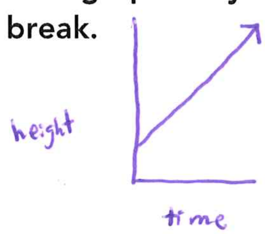
1. Consider your height from when you begin climbing after your break.

a. Define variables for the changing quantities and explain which is the independent quantity and which is the dependent quantity.

h = height
t = time

The height depends on the time so time is independent & height is dependent.

b. Sketch a graph for your journey up the cliff after the break.



c. Which quantities are changing? Which quantities remain constant?

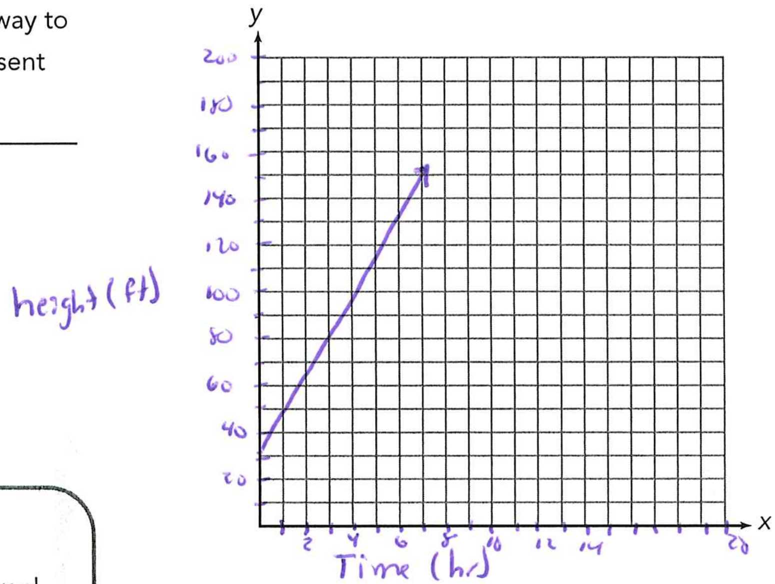
time & height change.
The rate you climb remains the same & the starting height remains the same.

d. Write an equation for the dependent quantity as a function of the independent quantity.

$h = 12t + 36$

Drawing a line through the data set of a graph is a way to model or represent relationships.

e. Create a graph to represent the situation. Label your axes appropriately.



f. State the domain and range for the situation.

Domain {All real #'s from 0 to 6}

Range {All real #s from 36 to 108}

g. Does this situation represent a function? Explain your reasoning.

yes

When you graph the input and output values of some functions, the graph forms a straight line. A function whose graph is a straight line is a **linear function.**

2. Consider what you know about linear relationships.

a. Is every line a linear function? Explain your reasoning.

No - vertical lines are not functions.

b. Is every linear function also a proportional relationship? Is every proportional relationship a linear function?

No - proportional lines go through (0,0)

yes - every proportional relationship is a linear function.

c. Describe how the independent and dependent values change in linear functions.

There is a constant ratio between the dependent & independent values in a linear function.

$\frac{\text{rise}}{\text{run}}$ $\frac{y}{x}$

d. Write the equation of a linear function with slope *m*, initial value *b*, independent quantity *x*, and dependent quantity *y*.

$y = mx + b$

3. Write an equation to model each linear function.

a. Lin is tracking the progress of her plant's growth. Today the plant is 5 centimeters high. The plant grows 1.5 centimeters per day. Write an equation that relates *h*, the height of the plant after *d* days.

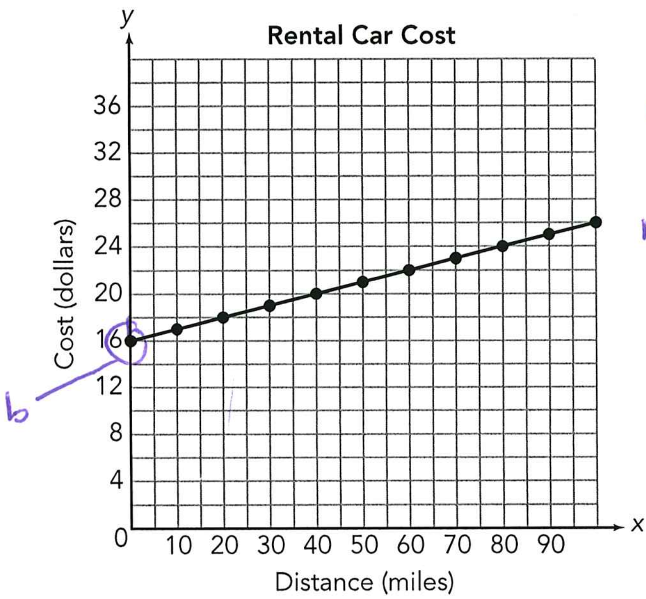
$h = 5 + 1.5d$

b. Carmen initially has money in her bank account. Each week she withdraws the same amount of money from her account. Write an equation that relates *b*, her account balance after *w* weeks.

Week	Account balance (dollars)
1	825
2	750
3	675
4	600

$b = 900 - 75w$

c. A rental car agency charges a fixed daily rate with an additional charge per mile driven. Write an equation that relates t, the total cost for a rental car, after m miles driven.



$m = \frac{rise}{run}$

$m = 4 \cdot \frac{2}{10} = \frac{1}{10}$

$y = mx + b$

$t = 16 + 0.1m$

d. Write an equation that relates y, the dependent quantity, to x, the independent quantity, if the slope is $\frac{2}{3}$ and the y-intercept is -7.

$y = mx + b$ $\quad$ $y = \frac{2}{3}x - 7$

ACTIVITY 4.2

Increasing, Decreasing, or Constant

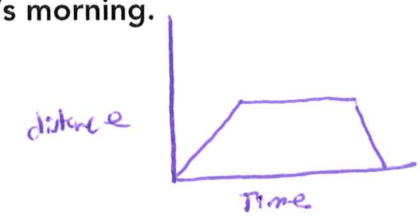
Saturday morning, Erika walked for 30 minutes at a steady rate from her house to a park 3 miles away. When she arrived, she played basketball for an hour, and then she caught a ride home with Kendall. They traveled at a constant speed from the park to Erika's house and arrived in 12 minutes.

1. Define variables for the time since Erika left home in minutes, and for her distance from home in miles.

t = time in minutes

d = distance in miles

2. Sketch a graph for Erika's morning.



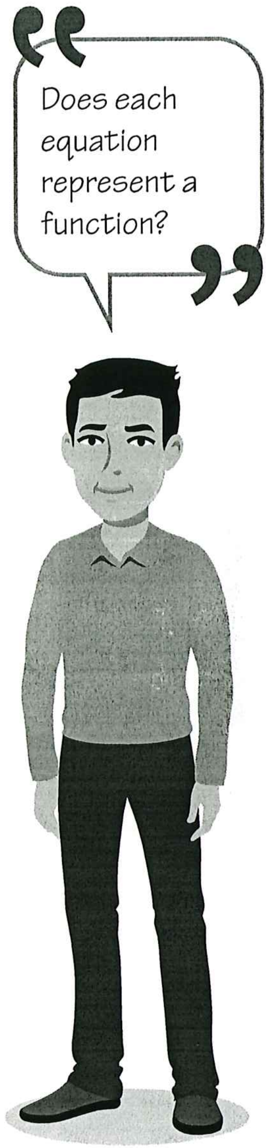
3. Determine the rate at which Erika walked to the park and the rate at which she and Kendall drove home. Express the rates in miles per minute.

$\frac{3 \text{ miles}}{30 \text{ min}}$ = 0.1 mile per min.

$\frac{3 \text{ miles}}{12 \text{ min}}$ = 0.25 miles per min.

4. Determine the domain (time) and range (distance from home) for each part of Erika's morning.

Activity	Domain	Range
Walking to Park	$0 \le t < 30$	$0 \le d < 3$
Playing Basketball	$30 \le t < 90$	$d = 3$
Riding Home	$90 \le t < 102$	$0 \le d \le 3$



5. **Write an equation that can be used to model each statement.**

   a. **Erika's distance from home as she walked to the park**

   $d = 0.1t$

   b. **Erika's distance from home while she was playing basketball**

   $d = 3$

   c. **Erika's distance from home as she rode home from the park**

   $d = 3 - 0.25t$

6. **Describe what happens to Erika's distance from home in each part her morning as time increases.**

   a. **Erika's walk to the park**

   As time increases, distance from home increases.

   b. **Playing basketball**

   As time increases, distance remains the same (constant)

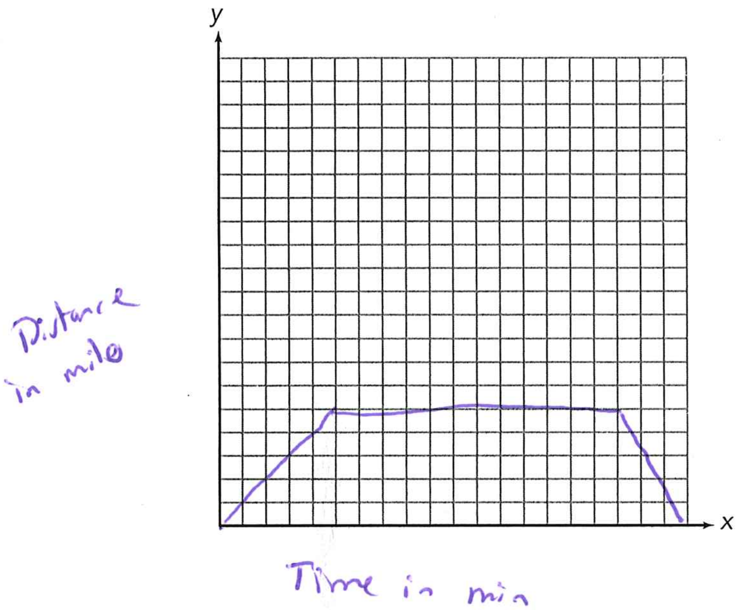
   c. **Erika's ride home**

   As time increases, her distance from home decreases.

You can describe a function by analyzing the values of the function.

- When both values of a function increase together, the function is called an **increasing function**.
- When the y-value of a function does not change, or remains constant, the function is called a **constant function**.
- When the value of a dependent variable decreases as the independent variable increases, the function is called a **decreasing function**.

7. Create a graph of Erika's morning. Label each axis.



8. Consider the graph of Erika's morning.

a. What are the domain and range for Erika's morning in this problem?

The domain is $0 \le t \le 102$

The range is $0 \le d \le 3$

b. Does the graph of Erika's morning represent a function? Explain your reasoning.

yes - she is only one distance from home at any given time.

c. List the parts of the function that are increasing, decreasing, or constant. Also list their equations and domains.

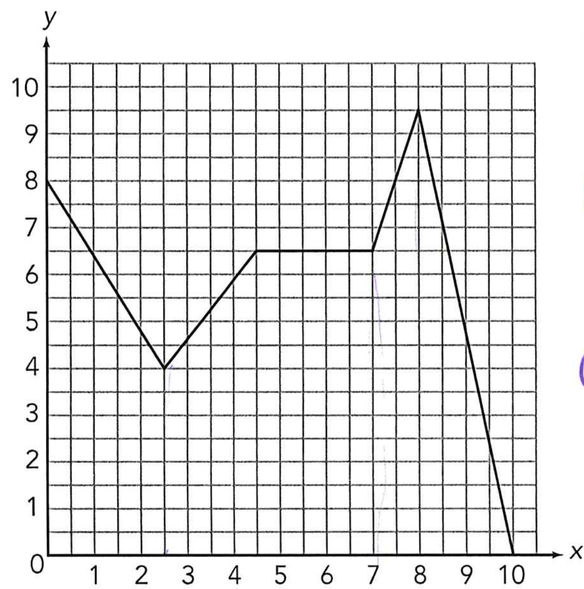
Activity	Behavior	Equation	Domain
Walking to Park	increasing	$d = 0.1t$	$0 \le t < 30$
Playing Basketball	Constant	$d = 3$	$30 \le t < 90$
Riding Home	decreasing	$d = 3 - 0.25t$	$90 \le t \le 102$

NOTES

You can describe the intervals of a function by analyzing what happens at specific independent values.

- When a function is increasing for some values of the independent variable, it is said to have an **interval of increase**.
- When a function is decreasing for some values of the independent variable, it is said to have an **interval of decrease**.
- When a function is constant for some values of the independent variable, it is said to have a **constant interval**.

9. Describe any intervals of increase, intervals of decrease, and constant intervals.



Increasing $2.5 \le x < 4.5$
$7 \le x < 8$

Decreasing $0 \le x < 2.5$
$8 \le x < 10$

Constant $4.5 \le x < 7$

10. Explain how the behavior of each part of the function relates to the slope for that part of the function.

Increasing = positive slope

Decreasing = negative slope

Constant = zero slope

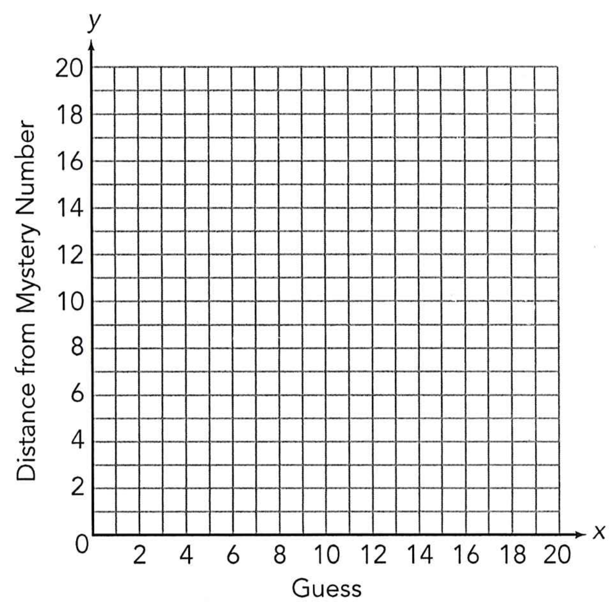
ACTIVITY **4.3**

Analyzing Non-Linear Functions

In this activity, have someone in your class think of a whole number from 1 to 20. One-by-one ask each of your classmates to guess what the number is. Then record each guess without revealing the mystery number.

1. **Record and analyze the results of each guess.**

   a. **After each guess, plot a point to represent the relationship between the value of the guess and its distance from the mystery number.**



   b. **What is the mystery number? Explain your reasoning.**

   c. **Does this graph represent a function? Explain your reasoning.**

   d. **Is this a linear function? Explain how you know.**

   e. **Identify the domain and range for this situation.**

   f. **Describe when the graph increases, decreases, or is constant.**

NOTES

The graph of the relationship between the value of the guess and its distance from the mystery number is an example of an *absolute value function*. Recall that the absolute value of a number is the distance from the number to zero on a number line. An **absolute value function** is a function that can be written in the form $y = |x|$, where x is any number or expression.

g. Write the equation that describes this absolute value function. How does the equation relate to the graph?

$y = |x - \#|$ # is the mystery #

Let's consider a different situation. Recall that the area of a square is equal to the side length, s, multiplied by itself and is written as $A = s^2$.

* You can't have a negative number for the length of the side of a square.

2. Use this relationship to answer each question.

a. What are the domain and range for $A = s^2$? Explain your reasoning. They are both the set of all real #s.

* any time you square a #, it becomes positive (a negative x a negative = positive)

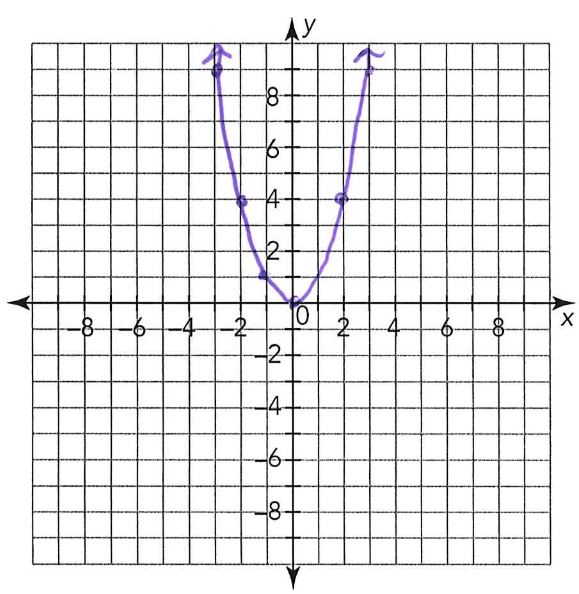
In the equation $A = s^2$, the side length of a square is the independent variable, and the area of the square is the dependent variable. This formula can also be modeled by the equation $y = x^2$.

b. What are the domain and range for $y = x^2$? How is this different from $A = s^2$?

$y = x^2$ Domain = ℝ Range = ℝ greater than 0.

$A = s^2$ Can only have positive side lengths so their domains are different.

c. Use the equation $y = x^2$ to complete the table of values. Then graph the values on the coordinate plane.



x	$y = x^2$
−3	9
−2	4
−1	1
−0.5	0.25
0	0
2	4
2.3	5.29
3	9

NOTES

d. Does this graph represent a function? Explain your reasoning.

yes - each input has a distinct output.
It also passes the vertical line test.

e. Is this a linear function? How do you know?

No - it's curved (U-shaped)

f. Describe when the graph increases, decreases, or is constant.

Decreases $x < 0$ and Increases $x > 0$

The graph of the relationship between the side length of a square and its area is an example of a *quadratic function*. A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where a, b, and c are any real numbers and a is not equal to zero.

Quadratic Equation has an x^2 term.

g. What are the values for a, b, and c in this equation? How does your equation fit the definition of a quadratic function?

$y = x^2$ $a = 1$ $b = 0$ $c = 0$

0 is a real # & $a \neq 0$.

Let's consider one more situation. Recall that the volume of a cube is equal to the side length, s, cubed and is written as $V = s^3$.

3. Use this relationship to answer each question.

a. What are the domain and range for $V = s^3$? Explain your reasoning.

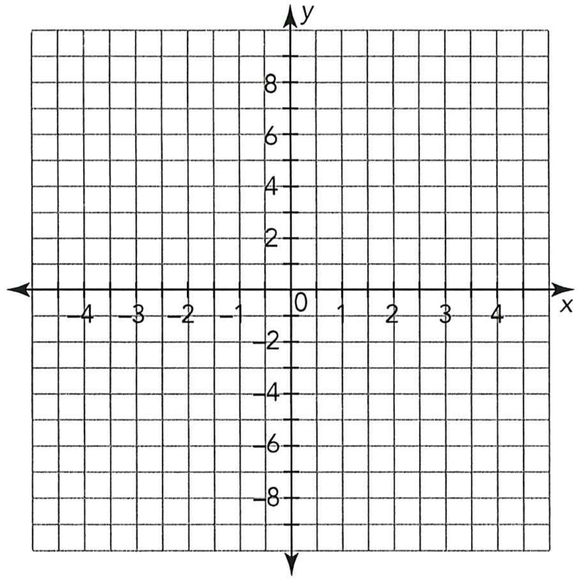
Domain & Range are all positive real #'s → you cannot have a negative side length or a negative volume.

In the equation $V = s^3$, the side length of a cube is the independent variable, and the volume of the cube is the dependent variable. This formula can also be modeled by the equation $y = x^3$.

b. What are the domain and range for $y = x^3$? How is this different from $V = s^3$?

Domain & Range are both all real #'s - you can have negative

c. Use the equation $y = x^3$ to complete the table of values. Then graph the values on the coordinate plane.



x	$y = x^3$
−2	−8
−1.5	−3.375
−1	−1
−0.5	−0.125
0	0
1.5	3.375
2	8

d. Does this graph represent a function? Explain your reasoning. yes - no x-values repeat. Each input has a distinct output.

e. Is this a linear function? How do you know?

No - it curves

f. Describe when the graph increases, decreases, or is constant. The graph is always increasing.

Cubic function has an x^3 term.

The graph of the relationship between the side length of a cube and its volume is an example of a *cubic function*. A **cubic function** is a function that can be written in the form $y = ax^3 + bx^2 + cx + d$, where each coefficient or constant a, b, c, and d is a real number and a is not equal to zero.

g. What are the values for a, b, c, and d in this equation? How does your equation fit the definition of a cubic function?

$a = 1$ $b = 0$ $c = 0$ $d = 0$ $a \neq 0$ & 0 is a real #.

NOTES

TALK the TALK

Show the Horse the Way

1. Sketch a graph for each set of given characteristics.

   a. increases over its entire domain

   b. decreases when $x < -2$ and increases when $x > -2$

   c. includes an interval of decrease, an interval of increase, and a constant interval

2. Write a possible story for the graph described in part (c).

NOTES

3. Write an equation for each function description.

a. a linear function

b. a decreasing function

c. a constant function

d. an increasing function

e. a decreasing and increasing function

Assignment

Write

Complete each sentence by writing the correct term or phrase from the lesson.

1. When the value of a dependent variable decreases as the independent variable increases, the function is said to be a(n) ______________.
2. When both the dependent and independent values of a function increase, the function is said to be a(n) ______________.
3. When a function is decreasing for some values of the independent variable, it is said to have a(n) ______________.
4. When the dependent variable does not change as the independent value of a function increases, the function is said to be a(n) ______________.
5. When a function is constant for some values of the independent variable, it is said to have a(n) ______________.
6. When a function is increasing for some values of the independent variable, it is said to have a(n) ______________.
7. When a function is a straight line that can be written in the form $y = mx + b$, it is said to be a(n) ______________.

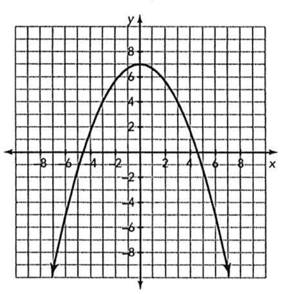
Remember

A function can be linear or non-linear, and functions are often represented using equations, graphs, and tables. Functions can be used to model everyday situations with specific domains.

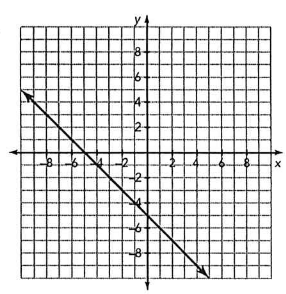
Practice

1. Create a table of values for each situation and identify the domain and range.
   a. Linear function
   b. Non-linear function
   c. Function that decreases and then increases
   d. Constant function
2. For each graph describe each interval of increase, interval of decrease, or constant interval.

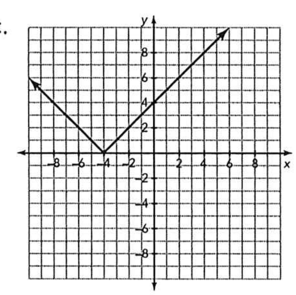
a.



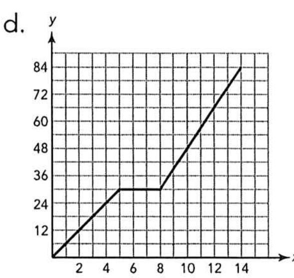
b.



c.



d.



3. When Randall wakes up Thursday morning, there are 15 inches of snow on the ground. The meteorologist reports that because the air temperature is slowly increasing, the snow will melt at a rate of 1.5 inches per day for the next 8 days. Then extremely cold temperatures over the following 3 days will prevent the snow from melting anymore. However, on day 11 of this streak of winter weather, the meteorologist predicts steady snow for the next 5 days, but only $\frac{1}{2}$ of an inch will accumulate per day. Let d represent the time in days since Thursday, and let h represent the height of the snow.

   a. Graph the function for the height of the snow over time.

   b. Describe each interval of increase, interval of decrease, or constant interval.

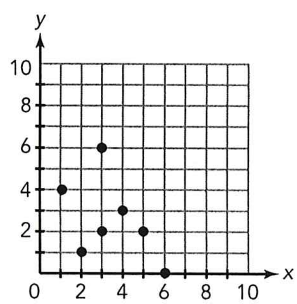
Stretch

Create the graph of a function that includes at least three of the following: a constant function, an absolute value function, a quadratic function, and a cubic function.

Review

1. State the domain and range of each relation. Then determine whether each is a function. Explain your reasoning.

   a.



   b. Sequence: 4, 14, 24, 34, 44, ...

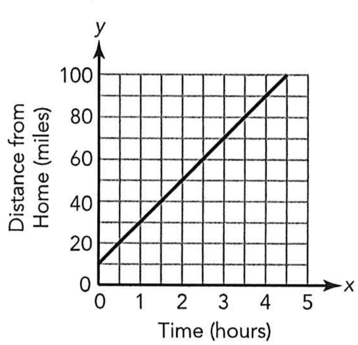
2. Graph each line using the information contained in the equation.

   a. $y = 2x - 5$

   b. $y - 4 = -\frac{3}{2}(x + 7)$

3. Use the graph shown to answer each question.

   a. What is the speed of the car in miles per hour?



   b. What is the cost of one snack?